

Calculus 1

Midterm Exam

October 1, 2025 (18:30 – 20:30)



university of
 groningen

Please read the instructions!

1) Prove using the (ε, δ) -definition that $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$.

2) Apply l'Hospital's rule to evaluate the following limit: $\lim_{x \rightarrow 0^+} \left(\frac{\tan x}{x} \right)^{1/x^2}$.
Indicate which rules of differentiation are being used in each step.

3) Determine the length of the longest ship that can turn the 90° corner at a T-shaped junction of two completely straight canals of width a and b . You may assume that the width of the ship is negligible (i.e. the ship is a line segment).

4) Apply the Mean Value Theorem to $f(x) = x \ln(x) - x$ prove the following

Theorem. If $0 < a < b$, then $\left(\frac{a}{b}\right)^b < \frac{e^a}{e^b} < \left(\frac{a}{b}\right)^a$.

5) Use Taylor series to find $f^{(2025)}(0)$ if $f(x) = x^{1675} \ln(x + 1)$.

6) Find the equation for the curve in the xy -plane that passes through the point $(1, -1)$ if the slope of its tangent line at x is always $3x^2 + 2$.

Instructions

- **write your name and student number on the top of each sheet of writing paper!**
- use the writing (lined) and scratch (blank) paper provided, raise your hand if you need more paper
- start each question on a new page
- use a pen with black or blue ink
- do not use any kind of correcting fluid or tape
- any rough work should be crossed through neatly so it can be seen
- this is a closed-book exam (books & notes are not permitted)
- you may use your own handwritten formula sheet (a 2-sided A4 paper) – the invigilators will check these
- you are also allowed to use a simple pocket calculator
- programmable/graphing calculators are not allowed
- your work should be clearly and logically structured
- **explain your reasoning using words**
- show all your calculations, an answer without any computation will not be rewarded
- each problem is worth 15 points
- upon completion¹ submit your worksheets at the front desk

¹At the end of the exam or after you finished whichever is sooner.